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Calculations of groundwater recessions caused by drainage wells in arid climate taking into account the non-uniform inversion of groundwater evaporation

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This article is the translation of the article "Rascety ponizenij urovnej gruntovyh vod ot dejstvia drenaznyh skvazin v usloviah aridnogo klimata s ucetom neodnorodnoj inversii isparenia gruntovyh vod [Расчеты понижений уровней грунтовых вод от действия дренажных скважин в условиях аридного климата с учетом неоднородной инверсии испарения грунтовых вод]" published on January 25, 2018.

Abstract

This article discusses dependencies to calculate the vertical drain wells taking into account evaporation inversion. In a zone near the well, where the depth of groundwater level (GWL) is below a critical level, the inversion is set as a constant value. In a zone, where GWL is above a critical level, evaporation inversion is linearly dependent on GWL depth which is unknown in advance. Flowing from underlying aquifers has been taken into account. To illustrate the importance of detailed accounting of evaporation inversion in Central Asian climatic conditions, the forecasted recessions of GWL were compared to those calculated using conventional methods. Calculations were made for climatic and hydrogeological conditions of the northern part of Bishkek.

Keywords: Inversion of evaporation, calculations of vertical drainage, critical depth of groundwater level

Inversion of groundwater evaporation is the most important factor to be taken into account in hydrogeological calculations for arid climate. When groundwater levels (GWL) fall below the critical level, the volume of additional water due to reduced evaporation may reach 5-8 ths m^3/ha per year and more. Ignoring this factor in the calculations of vertical drainage leads to significant errors in forecasted GWL depths (in the northern part of Bishkek, the error is 1-2 m). Critical depth of GWL for the Chuy Valley ranges from 2.5 to 3.0 m. In most of studies [1], evaporation inversion is set as a constant value, i.e. does not depend on GWL depth. This leads to underestimated impact of drain wells on GWL. Appropriate comparative assessment is made in this article. A number of studies provide various types of spatial averaging of evaporation inversion [2].

Representation of evaporation inversion depending on unknown-in-advance GWL depth recessions which is acceptable for arid climate, is given below.

$$W = \frac{Uo}{Skr} \cdot S \quad \text{given} \quad 0 \leq S \leq Skr \tag{1}$$

$$W = Uo \quad \text{given} \quad S > Skr \tag{2}$$

where W – groundwater evaporation inversion, m/day;

Uo – evaporation of groundwater with initial GWL position, m/day;

S = S(r) sought-for GWL recessions, m;

r – distance to a drain well, m;

Skr – recession of initial GWL up to a critical level, m.

If GWL does not drop below a critical level, then the dependence (1) is applied in the entire filtration zone, and there are no principal difficulties in a solution of the relevant filtration equations. There will be no problems, if GWLs are below the critical level everywhere. In this case, the dependence (2) is applied in the entire filtration zone. However, the most common case is when GWLs are below the critical level in the zones adjacent to wells, and above the critical level – beyond such zones. Boundary position between the zones is not known in advance.

Mathematical formulation of the well problem, considering the flowing and groundwater evaporation inversion, based on the dependencies (1), (2), is as follow [3, 4] Figure 1:

$$\frac{d^2}{dr^2} S1(r) + \frac{1}{r} \cdot \left(\frac{d}{dr} S1(r) \right) - \frac{S1(r)}{B^2} - \frac{W}{T} = 0 \quad \text{given} \quad 0 < r \leq R \tag{3}$$

$$\frac{d^2}{dr^2} S2(r) + \frac{1}{r} \cdot \left(\frac{d}{dr} S2(r) \right) - \frac{S2(r)}{P^2} = 0 \quad \text{given} \quad r \geq R \tag{4}$$

where S1(r) – GWL recessions in the zone $0 < r \leq R$, m;

S2(r) – GWL recessions in the zone $r \geq R$, m.

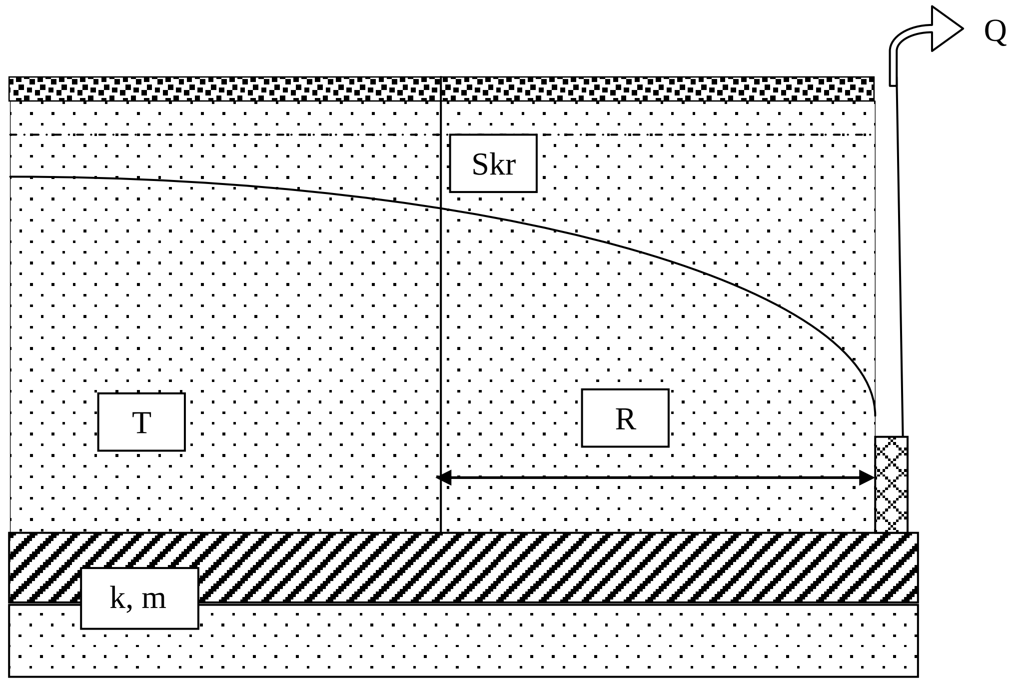

Figure 1. Calculation scheme

R – distance from the well to the boundary of zone with GWL above a critical level, m;
r – distance to the well, m;
T – transmissibility of aquifer, m^2/ day;
W – groundwater evaporation inversion, m/day.
m, k – filtration capacity and coefficient of underlying low-permeability layer m, m/day.

$$B = \sqrt{\frac{T \cdot m}{k}} \qquad P = \sqrt{\frac{T \cdot m}{k + a \cdot m}} \qquad a = \frac{Uo}{Skr}$$

To solve equations (1) and (2) the following five boundary conditions are used. Conventional formulation of the problem consisting of 2 second-order ordinary differential equations, requires four boundary conditions. However, in this particular case, the boundary location is also unknown, which requires additional condition.

Condition on the "wall" of well:

$$\lim_{r \to 0} \left(2 \cdot \pi \cdot r \cdot T \cdot \frac{d}{dr} S1(r) \right) = -Q \tag{5}$$

where Q – flow of the well, m^3/day.

Boundary conditions r = R:

$$\frac{d}{dr}S1(R) = \frac{d}{dr}S2(R) \tag{6}$$

$$S1(R) = S2(R) \tag{7}$$

$$S1(R) = Skr \tag{8}$$

Infinity condition:

$$\lim_{r \to \infty} S2(r) = 0 \tag{9}$$

Overall solution of the differential equation system (1), (2) is expressed by a linear combination of cylinder functions of an imaginary argument [5, 6] with an additional item associated with auxiliary power:

$$S1(r) = C1(R) \cdot I0\left(\frac{r}{B}\right) + C2(R) \cdot K0\left(\frac{r}{B}\right) - \frac{B^2 \cdot W}{T} \tag{10}$$

$$S2(r) = C3(R) \cdot I0\left(\frac{r}{P}\right) + C4(R) \cdot K0\left(\frac{r}{P}\right) \tag{11}$$

where I0(x), K0(x) – zeroth-order cylinder functions of the imaginary argument of the zero order of the first and second kind, respectively;
C1(R), ..., C4(R) – arbitrary constants that depend on R (position of the boundary between zones). They are defined by the ratios (5) - (9) using the properties of cylinder functions [7]. Below are derived ratios for C1 – C4; intermediate calculations are omitted for brevity.

$$C2 = \frac{Q}{2 \cdot \pi \cdot T} \tag{12}$$

$$C3 = 0 \tag{13}$$

$$C1(R) = \frac{C2\cdot\left[\left(\frac{1}{P}\cdot\frac{K0\left(\frac{R}{B}\right)}{K0\left(\frac{R}{P}\right)}\cdot K1\left(\frac{R}{P}\right)\right) - \frac{1}{B}\cdot K1\left(\frac{R}{B}\right)\right]}{\frac{1}{P}\cdot\frac{I0\left(\frac{R}{B}\right)\cdot K1\left(\frac{R}{P}\right)}{K0\left(\frac{R}{P}\right)} + \frac{1}{B}\cdot I1\left(\frac{R}{B}\right)} \quad (14)$$

where I1(x), K1(x) – first-order cylinder functions of the imaginary argument of the first and second kind, respectively;

$$C4(R) = C1(R)\cdot\frac{I0\left(\frac{R}{B}\right)}{K0\left(\frac{R}{P}\right)} + C2\cdot\frac{K0\left(\frac{R}{B}\right)}{K0\left(\frac{R}{P}\right)} - \frac{B^2\cdot\frac{W}{2}}{T\cdot K0\left(\frac{R}{P}\right)} \quad (15)$$

R is defined from the equation (8) using expressions (12) – (15). The equation (8) is not solved using conventional methods, it is easy to be solved graphically with any accuracy by using the Mathcad 2001i computer system. For convenience, the equation is given as follows:

$$F(R) = S1(R) - Skr = 0 \quad (16)$$

For 80 m deep drain well with flow rate of 40 l/s in the northern part of Bishkek, F(R) is given in Figure 2. Graph plotting is automated, the solution to equation (8) is straightforward (R = 50 m).

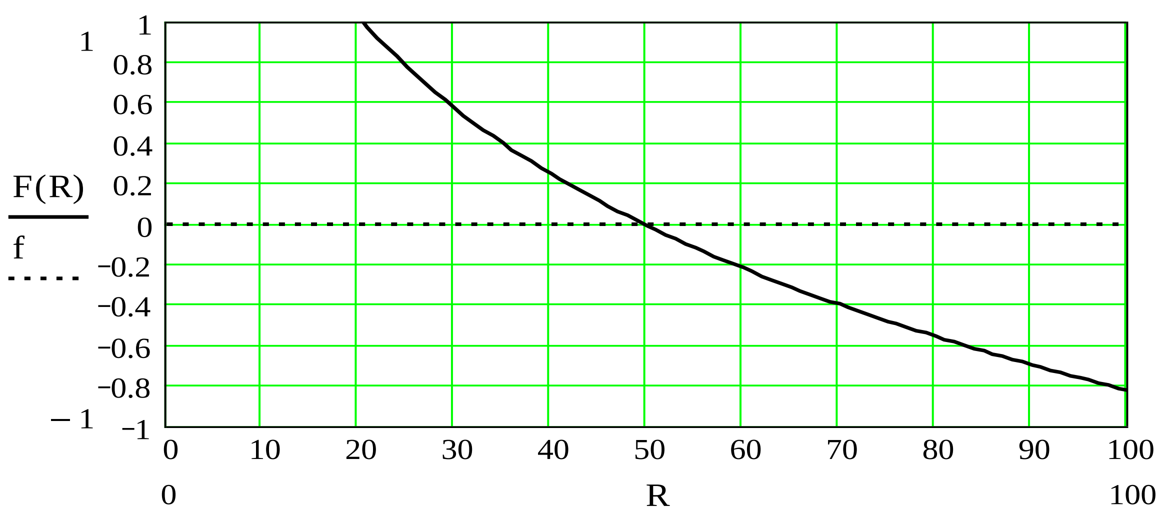

Figure 2. F(R) graph to solve the equation F(R) = 0

Based on the found R value, the arbitrary constants C1(R) and C4(R) are determined, then S1(r) and S2(r) are determined. For the territory considered in this article, the found function of GWL recessions caused by a single well is given in Figure 3. This figure also shows the recessions curve calculated without regard to evaporation inversion, and the recessions curve with evaporation inversion set as a constant. The difference in forecasted GWL depths is more than 1 m, which suggests that it is impossible to use conventional formulas for calculation of vertical drainage in Central Asian climatic conditions.

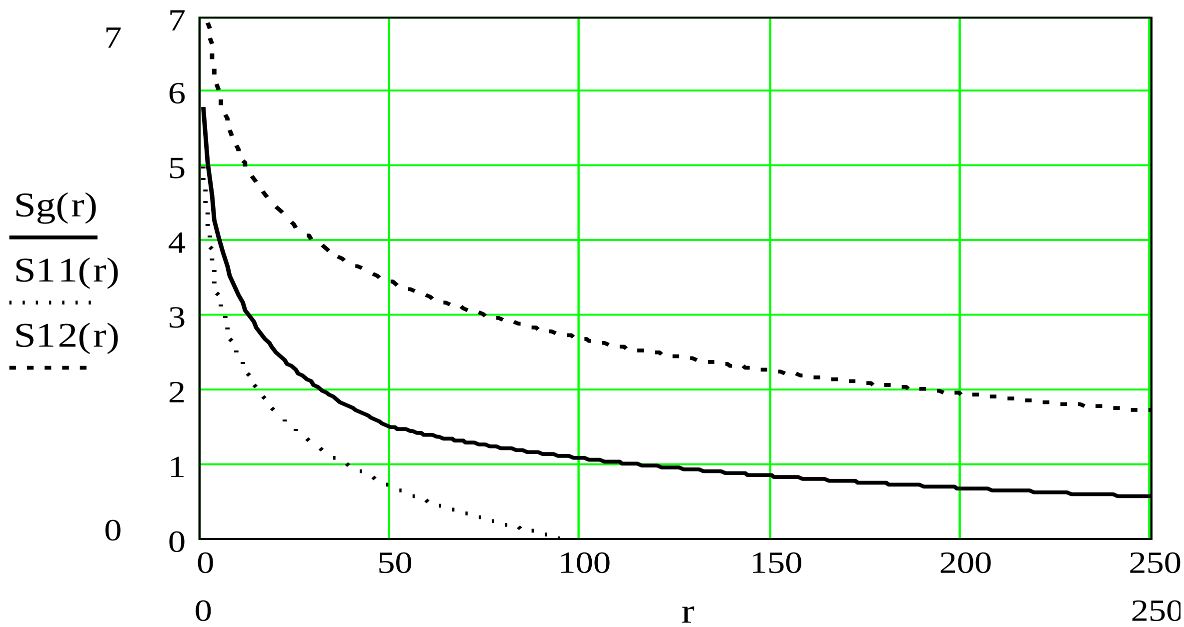

Figure 3. Estimated recessions under the influence of a drain well in the northern part of Bishkek (Sg – recommended solution, S11 – evaporation inversion does not depend on GWL depth, S12 – evaporation inversion is not taken into account)

The method proposed in the article can significantly improve the accuracy and reliability of vertical drainage calculations for different areas in arid climate conditions.

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